CCC Annual Report

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Constitutive Equations for Steel at Elevated Temperatures

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Executive Summary

- This work investigates new and currently used constitutive equations for the inelastic mechanical behavior of steel from 400°C-1600°C.

Key Findings:

- Model V, with a separately evolving back stress term, performs much better than Model III in cyclic loading.
- Kozlowski's Model III for austenite performs well in cyclic loading while also matching experimental tensile test, stress relaxation, and creep data.

Introduction

- Constitutive laws to accurately describe the inelastic flow of steel in cyclic loading are important for FEM models of continuous casting.
- There are four common types of constitutive models: time-independent elastoplastic, elastoplastic with creep, unified models with evolving state variables, and elasto-viscoplastic models.
- Two new constitutive equations to describe the inelastic mechanical behavior of steel from 400°C-700°C were developed.

Current Constitutive Models Used by CCC Researchers

- Liquid (Elastic-Perfectly Plastic):
  - $\dot{\varepsilon}_{ie} = \dot{\varepsilon}_T$ IF $\varepsilon_T > (\sigma_y)\varepsilon_E$
- Ferrite (Zhu Power Law):
  - $\dot{\bar{\varepsilon}}_{ie} = 0.1 F_\delta |F_\delta|^{0.1}$
  - $F_\delta = \frac{c\bar{\sigma}}{f_C (T(K)/300)^{-5.52} (1+1000|\bar{\varepsilon}_{ie}|)^m}$
  - $m = -9.4156\ X\ 10^{-5}\ T(K) + 0.349501$
  - $n = (1.617\ X\ 10^{-4}\ T(K) - 0.6166)^{-1}$
- Austenite (Kozlowski Model III):
  - $\dot{\bar{\varepsilon}}_{ie} = C \exp\left(\frac{-4.465\ X\ 10^4}{T(K)}\right) F_\gamma |F_\gamma|^{n-1}$
  - $F_\gamma = c\bar{\sigma} - a_\varepsilon \bar{\varepsilon}_{ie} |\bar{\varepsilon}_{ie}|^{n_\varepsilon - 1}$
  - $a_\varepsilon = 130.5 - 5.128\ X\ 10^{-3}\ T(K)$
  - $n_\varepsilon = -0.6289 + 1.114\ X\ 10^{-3}\ T(K)$
  - $n = 8.132 - 1.54\ X\ 10^{-3}\ T(K)$
  - $C = 4.655\ X\ 10^4 + 7.14\ X\ 10^4 (pct\ C) + 1.2\ X\ 10^5 (pct\ C)^2$

Previous Work

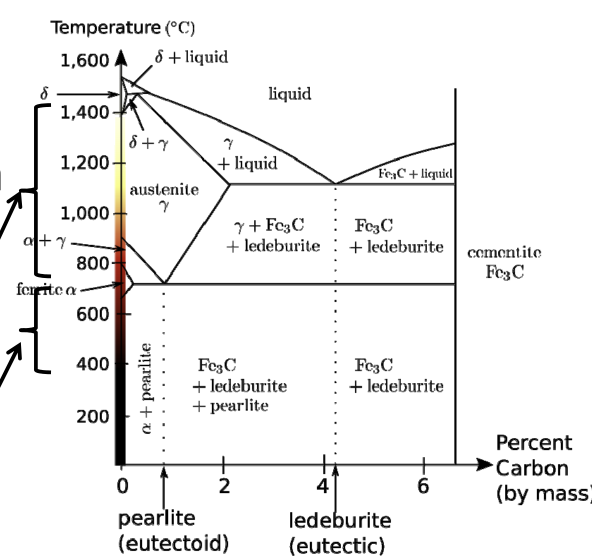
- In Kozlowski's[1] paper, four constitutive equations were developed to model the inelastic flow behavior of austenitic steel.
- These four constitutive equations were then evaluated through comparison of the model's behavior with experimental data of steel from Wray[3] and Suzuki[2].
- Kozlowski's best model, Model III, was an elasto-viscoplastic equation with a back stress calculated using a temperature dependent constant and inelastic strain.
- In Lu's[6] paper, a unified model with evolving state variables was developed with a back stress term that evolves depending on the inelastic strain. This model was developed for cyclic loading.

New Work

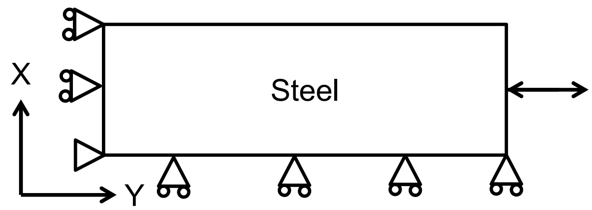
- A constitutive equation to describe the inelastic flow of steel in the ferrite + cementite phase region was developed.

Temperature range of Kozlowski III model.

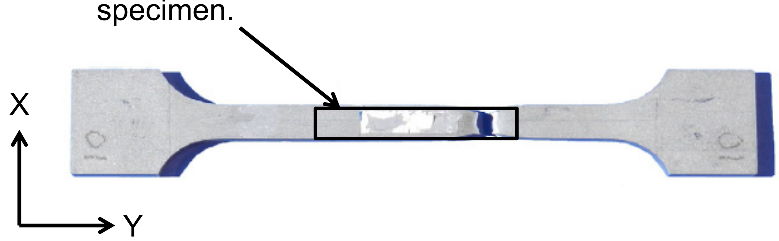
Temperature range of new model.



Model Domain



Location of model domain on experimental specimen.



Model Equations

- Model III (from Kozlowksi):
  - $\dot{\varepsilon}_{ie} = C \exp\left(\frac{-Q}{T}\right) \left[\sigma - a_{\varepsilon} {\varepsilon_{ie}}^{n_{\varepsilon}}\right]^{n}$

Elasto-viscoplastic type model. No separately evolving state variables.

- Model V (from Lu):
  - $\dot{\varepsilon}_{ie} = C \exp\left(\frac{-Q}{T}\right) \left[\sigma - \alpha\right]^{n}$
  - $\dot{\alpha} = g\left(\dot{\varepsilon}_{ie} - \frac{\alpha}{\alpha^{*}} \left|\dot{\varepsilon}_{ie}\right|\right)$

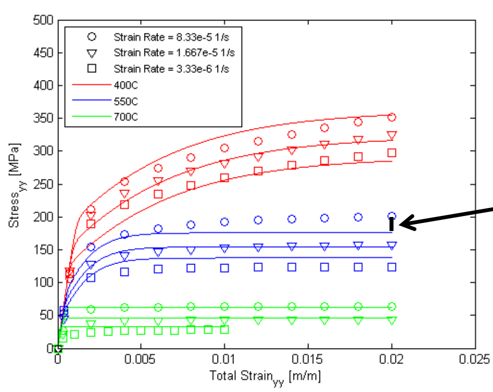
Unified type model with a separately evolving state variable.
Extra calculations necessary to evolve state variable.

Equation Solution

- To solve for the mechanical behavior of the steel, the inelastic strain rate equation is integrated over time.
- The stress, elastic, and inelastic strains were then calculated from the below equations:
  - $\dot{\varepsilon} = \dot{\varepsilon}_e + \dot{\varepsilon}_{ie}$
  - $\varepsilon_{t+\Delta t} = \varepsilon_t + \dot{\varepsilon}\,\Delta t$
  - $\sigma = E\varepsilon_e$

Model Calibration

- The free parameters in each model were then optimized using the Downhill Simplex Method to minimize differences between model predicted stresses and measured stresses from Knobloch[5].
- Stress difference calculated between constitutive model and experimental data at 11 total strain values.
  - $Error(MPa) = \frac{\sqrt{\sum(\sigma_{model} - \hat{\sigma}_{measured})^2}}{N}$ N = Total number of data points



Error for one data point. Total error computed, summed and averaged for all 99 points.

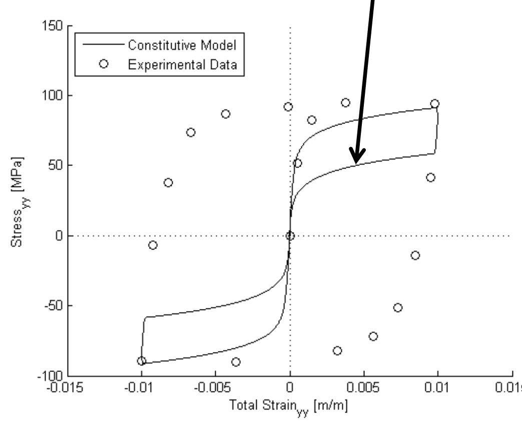
Model Temperature Dependence

- Model III (from Kozlowksi):
  - $\dot{\varepsilon}_{ie} = C \exp\left(\frac{-Q}{T}\right)[\sigma - a_\varepsilon {\varepsilon_{ie}}^{n_\varepsilon}]^{n1}$
- Model V (from Lu):
  - $\dot{\varepsilon}_{ie} = C \exp\left(\frac{-Q}{T}\right)[\sigma - \alpha]^{n2}$
  - $\dot{\alpha} = g\left(\dot{\varepsilon}_p - \frac{\alpha}{\alpha^*}|\dot{\varepsilon}_p|\right)$
- Parameters highlighted in yellow are temperature dependent.
- Form of temperature dependence for Model III parameters:
  - $n1 = A + B(T)$
  - $n_\varepsilon = C + D(T)$
  - $a_\varepsilon = E + F(T) + G(T^2)$
- Form of temperature dependence for Model V parameters:
  - $n2 = H + L(T)$
  - $g = M + N(T)$
  - $\alpha^* = P + R(T) + S(T^2)$

Need for New Model

- $\dot{\varepsilon}_{ie} = C \exp\left(\frac{-Q}{T}\right)[\sigma - a_\varepsilon {\varepsilon_{ie}}^{n_\varepsilon}]^{n}$
- Evolution of back stress term $a_\varepsilon {\varepsilon_{ie}}^{n_\varepsilon}$ in Model III is not appropriate in unloading. Back stress term is too large in unloading, evolving with strain inappropriately.
- Shape of unloading curve will be the same shape as loading curve with Model III.

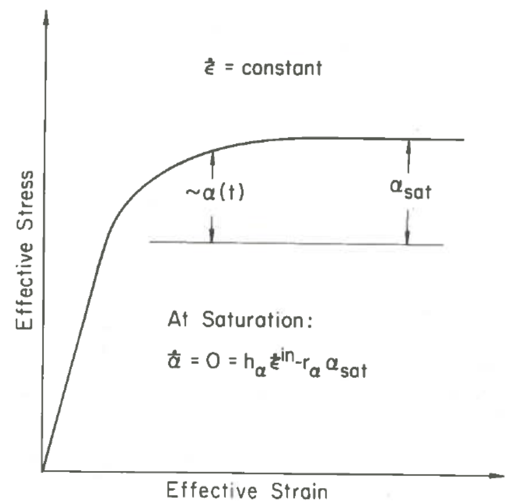
Poor unloading behavior



Initial fit of Model III to uniaxial tensile data lead to poor cyclic behavior. Cyclic data from Slavik[4].

Development of New Model

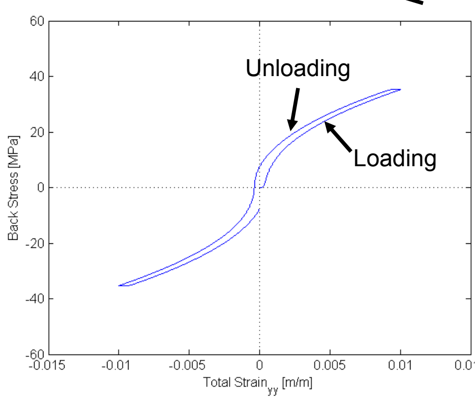
- Back stress appears to reach a saturation value in uniaxial tension.
- Lu[6] proposed a form for back stress (α) evolution:
  - $\dot{\alpha} = g\left(\dot{\varepsilon}_{ie} - \frac{\alpha}{\alpha^*}|\dot{\varepsilon}_{ie}|\right)$
- The back stress exponentially approaches a saturation value α^*.



From Slavik[4]. Evolution of the back stress with respect to effective strain for steel at 20°C.

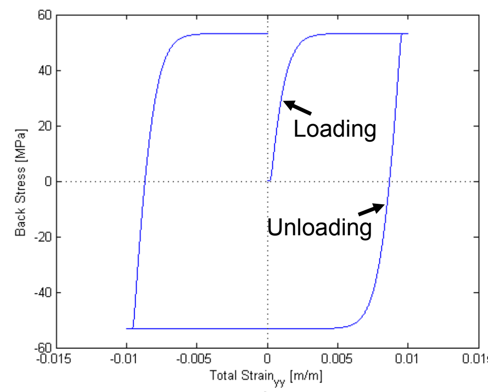
Back Stress Evolution Comparison

$\dot{\varepsilon}_{ie} = C\exp\left(\frac{-Q}{T}\right)[\sigma - \alpha_{\varepsilon}\varepsilon_{ie}^{\,n_{\varepsilon}}]^n$ Back stress



Model III, evolution of back stress term during cyclic loading.

$\dot{\varepsilon}_{ie} = C\exp\left(\frac{-Q}{T}\right)[\sigma - \alpha]^n,\ \dot{\alpha} = g\left(\dot{\varepsilon}_{ie} - \frac{\alpha}{\alpha^*}|\dot{\varepsilon}_{ie}|\right)$ Back stress



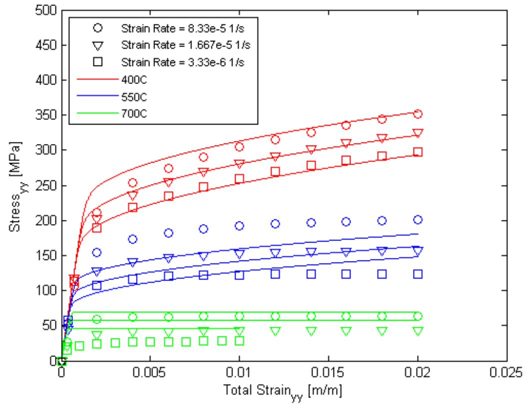
Model V, evolution of back stress term during cyclic loading.

- Model III's back stress remains too large in unloading, causing poor unloading behavior. Model V's back stress evolves similarly in initial loading and unloading.

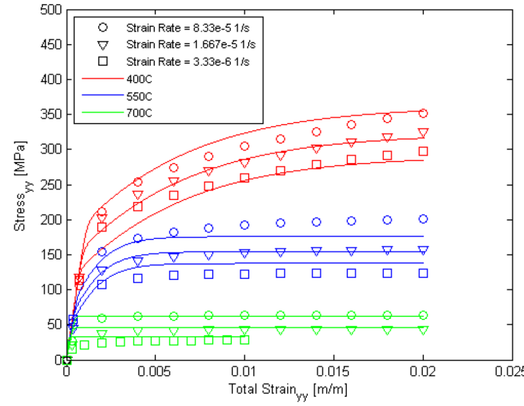
Model Evaluation Results

(Uniaxial Tension)

Model III



Model V



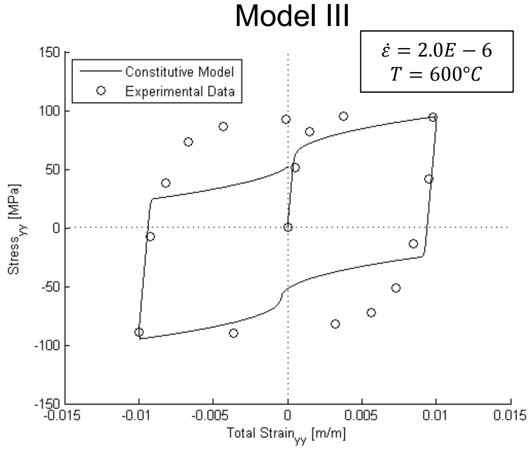
Model	Error [MPa]
Model III	1.10
Model V	1.55

- Model III shows a poorer fit at higher temperatures.
- Model V is a better match at high temperatures and small strains.
- Experimental data from Knobloch[5].

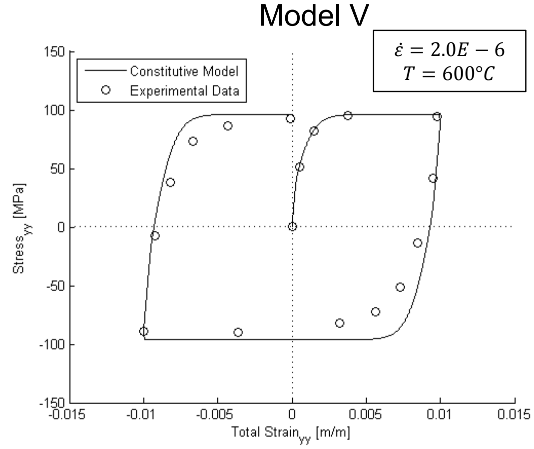
Model Evaluation Results

(Cyclic Loading)

Model III



Model V

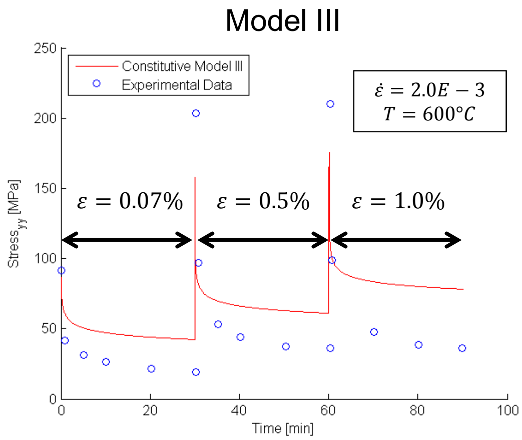


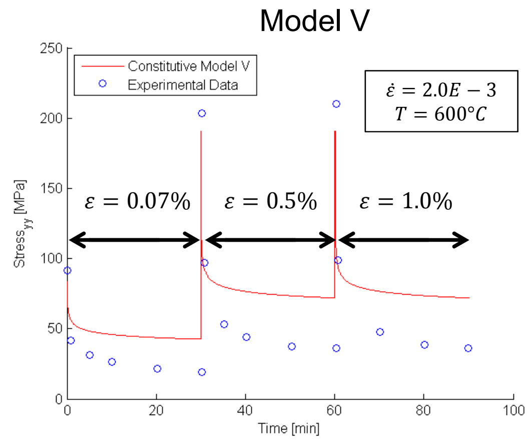
Model	Error [MPa]
Model III	11.2
Model V	10.2

- Model III was fit to cyclic data but is still a poor fit.
- Model V was not fit to cyclic data but is a better match.
- Experimental data from Slavik[4].

Model Evaluation Results

(Stress Relaxation)



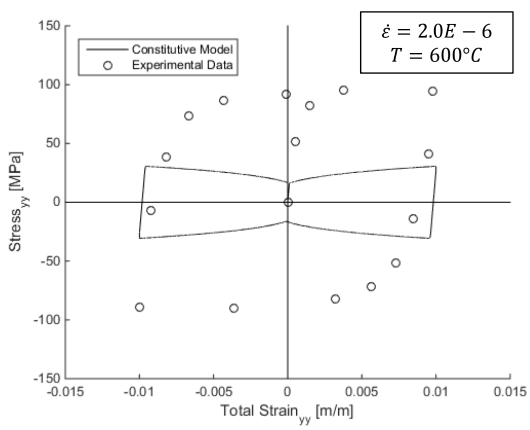


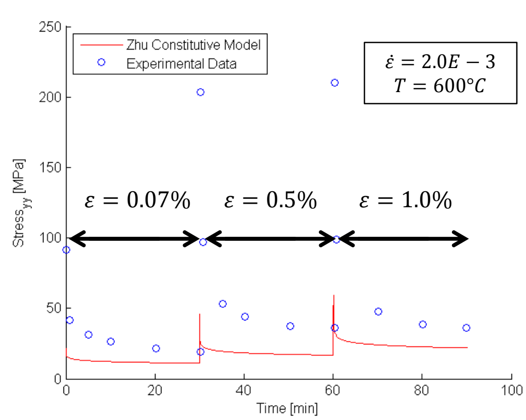
Model	Error [MPa]
Model III	15.2
Model V	15.2

- Model III better matches stresses during the stress relaxation.
- Model V better matches peak stresses after extension.
- Experimental data from Slavik[4].

Current Ferrite Model

(Zhu Power Law)

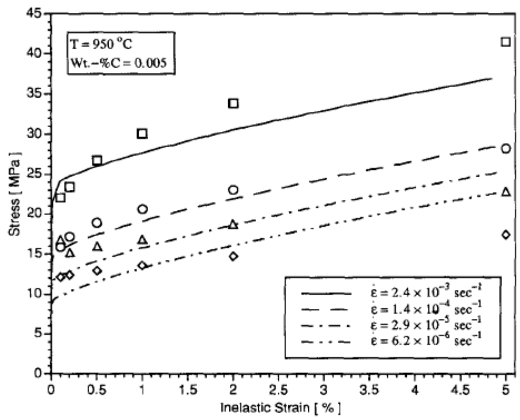




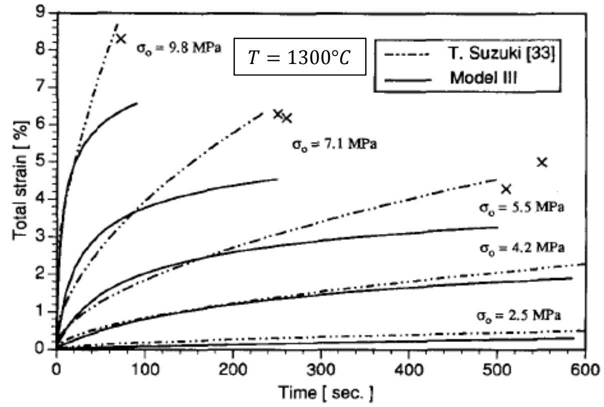
- Zhu power law model has a poor shape in cyclic loading, and seriously under predicts stresses for this steel grade.
- Zhu power law model is not capable of matching cyclic behavior of alpha-ferrite well because of the absolute value on effective inelastic strain.
- $\dot{\varepsilon}_{ie} = 0.1\left(\frac{c\bar{\sigma}}{f_c(T(K)/300)^{-5.52}(1+1000|\bar{\varepsilon}_{ie}|)^m}\right)^n$

Current Austenite Model

(Experimental Match)



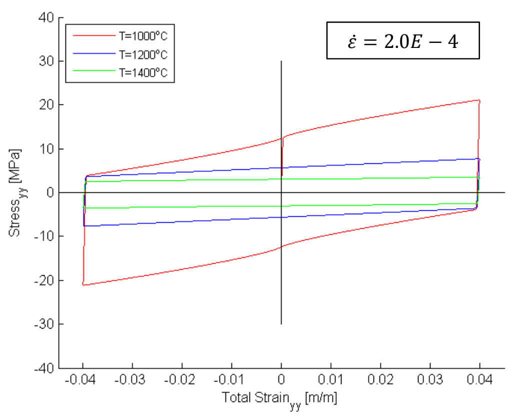
Comparison of Model III for austenite with tensile test data.



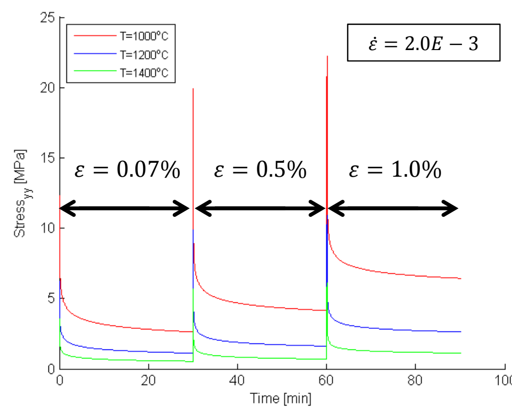
Comparison of Model III for austenite with creep test data.

- Kozlowski's Model III for austenite reasonably matches tensile test data from Wray[3] and creep test data from Suzuki[2].

Current Austenite Model



Cyclic loading of Kozlowski Model III at various temperatures.

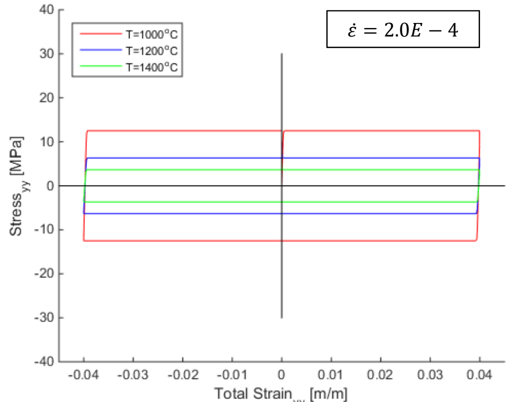


Stress relaxation of Kozlowski Model III at various temperatures.

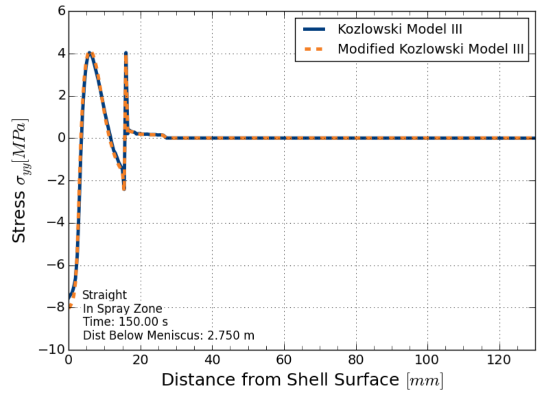
- Kozlowski Model III for austenite shows reasonable behavior in cyclic loading and for stress relaxation.
- Experimental data for austenite in cyclic loading is not currently available.

Current Austenite Model

(No Hardening Version)



Cyclic loading of **Modified** Kozlowski Model III at various temperatures.



Stress during solidification of **Modified** Kozlowski Model III.

- No-Hardening Kozlowski Model III has no back stress term:
  - $\dot{\varepsilon}_{ie} = C\exp\left(\frac{-Q}{T}\right)[\sigma - \cancel{a_\varepsilon \varepsilon_{ie}{}^{n_\varepsilon}}]^n$
- No-Hardening Kozlowski Model III shows a very close match with Kozlowski Model III in solidification models.

Conclusions & Future Work

- Two new constitutive equations have been developed and validated for ferrite + cementite from 400°C-700°C.
- Model V is better than Model III because it more closely matches uniaxial tension, cyclic loading and stress relaxation tests.
- Kozlowski's Model III for austenite performs well, including cyclic loading.
- The No-Hardening Kozlowski Model III for austenite performs very similarly to the original austenite Model III when used to model solidification.
- Zhu's power law for ferrite does not perform well in cyclic loading for alpha-ferrite.
- Model V should be expanded to fit different grades of steel and then implemented in the Abaqus UMAT.

Acknowledgments

- Continuous Casting Consortium Members (ABB, AK Steel, ArcelorMittal, Baosteel, JFE Steel Corp., Magnesita Refractories, Nippon Steel and Sumitomo Metal Corp., Nucor Steel, Postech/ Posco, SSAB, ANSYS/ Fluent)
- Professor Brian Thomas, Dr. Lance C. Hibbeler

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5. Knobloch, Markus, Jacqueline Pauli, and Mario Fontana. "Influence of the strain-rate on the mechanical properties of mild carbon steel at elevated temperatures." *Materials & Design* 49 (2013): 553-565.
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